P.R. GOVT. COLLEGE (AUTONOMOUS), KAKINADA
II year B.Sc., Degree Examinations - IV Semester
Mathematics Course-IV: REAL ANALYSIS
(w.e.f. 2020-21 Admitted Batch)

Total Hrs. of Teaching-Learning: 75 @ 6 hr/Week **Total credits: 04**

OBJECTIVES:

- Be able to understand and write clear mathematical statements and proofs.
- Be able to apply appropriate method for checking whether the given sequence or series is convergent.
- Be able to develop students ability to think and express themselves in a clear logical way.

This curriculum gives an opportunity to learn about the derivatives of functions and its applications.

Course Outcomes:

After successful completion of this course, the student will be able to

- get clear idea about the real numbers and real valued functions.
- obtain the skills of analyzing the concepts and applying appropriate methods for testingconvergence of a sequence/ series.
- Test the continuity and differentiability and Riemann integration of a function.
- Know the geometrical interpretation of mean value theorems.

UNIT I (12 Hours)

Introduction of Real Numbers (No question is to be set from this portion)

Real Sequences: Sequences and their limits, Range and Boundedness of Sequences, Limit of a sequence and Convergent sequence. The Cauchy's criterion, properly divergent sequences, Monotone sequences, Necessary and Sufficient condition for Convergence of Monotone Sequence, Limit Point of Sequence, Subsequences , Cauchy Sequences – Cauchy's general principle of convergence theorem.

UNIT II: (12 Hours)

INFINITIE SERIES :

Series : Introduction to series, convergence of series. Cauchy's general principle of convergence forseries tests for convergence of series, Series of Non-Negative Terms.

1. P-test
2. Cauchy's n^{th} root test or Root Test.
3. D'-Alemberts' Test or Ratio Test.
4. Alternating Series – Leibnitz Test.

UNIT III: (12 Hours)

CONTINUITY:

Limits: Real valued Functions, Boundedness of a function, Limits of functions. Some extensions of the limitconcept, Infinite Limits. Limits at infinity. (No question is to be set from this portion).

Continuous functions: Continuous functions, Combinations of continuous functions, Continuous Functionson interval.

UNIT IV: **(12 Hours)**

DIFFERENTIATION AND MEAN VALUE THEOREMS: The derivability of a function, on an interval, at a point, Derivability and continuity of a function, Graphical meaning of the Derivative, Mean value Theorems; Rolle's Theorem, Lagrange's Theorem, Cauchy's Mean value Theorem

UNIT V: **(12 Hours)**

RIEMANN INTEGRATION : Riemann Integral, Riemann integral functions, Darboux theorem. Necessary and sufficient condition for R – integrability, Properties of integrable functions, Fundamental theorem of integral calculus, First mean value Theorem.

Co-Curricular Activities(15 Hours)

Seminar/ Quiz/ Assignments/ Real Analysis and its applications / Problem Solving.

TEXT BOOK:

1. Introduction to Real Analysis by Robert G.Bartle and Donlad R. Sherbert, published by JohnWiley.

REFERENCE BOOKS:

1. A Text Book of B.Sc Mathematics by B.V.S.S. Sarma and others, published by S. Chand &Company Pvt. Ltd., New Delhi.
2. Elements of Real Analysis as per UGC Syllabus by Shanthi Narayan and Dr. M.D. Raisinghania,published by S. Chand & Company Pvt. Ltd., New Delhi

BLUE PRINT FOR QUESTION PAPER PATTERN
SEMESTER-IV

Unit	TOPIC	S.A.Q	E.Q	Marks allotted to the Unit
I	**Real Sequences**	2	2	30
II	**Infinite Series**	2	2	30
III	Continuity	2	2	30
IV	Differentiation And Mean value theorems	1	2	25
V	Riemann Integrations	1	2	25
Total		8	10	140

S.A.Q. = Short answer questions (5 marks)

E.Q = Essay questions (10 marks)

Short answer questions : 4 X 5 = 20

Essay questions : 4 X 10 = 40

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Total Marks = 60

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P.R. Government College (Autonomous), Kakinada
II Year, B.Sc., Degree Examinations - IV Semester
Mathematics Course: Real Analysis
Paper–IV (Model Paper ((w.e.f. 2020-21 Admitted Batch)

Time: 2Hrs 30 min **Max. Marks: 60**

PART-I

Answer any FOUR questions. Each question carries FIVE marks. **4 X 5 M=20 M**

1. Show that $\lim_{n\to\infty}\left[\sqrt{\frac{1}{n^2+1}}+\sqrt{\frac{1}{n^2+2}}+\cdots+\sqrt{\frac{1}{n^2+n}}\right]=1.$
2. Prove that every convergent sequence is a Cauchy sequence
3. If $\sum u_n$ convergences absolutely then prove that $\sum u_n$ converges.
4. Test for the convergence of $\sum_{n=1}^{\infty}\frac{1.3.5\ldots(2n-1)}{2.4.6\ldots 2n}x^{n-1}\ (x>0)$
5. Examine for continuity the function f defined by $f(x)=\ |x|+|x-1|$ at 0 and 1
6. Prove that $f: R\to R$ given by $f(x)=x^2$ is a continuous function on R, but not uniformly continuous on R
7. Show that $f(x)=x\ sin\left(1/_{x}\right), x\neq 0; f(x)=0, x=0$ is continuous but not derivable at $x=0$.
8. State and prove fundamental theorem of Integral Calculus.

PART - II

Answer ALL questions. Each question carries Eight marks. **5 X 8 M = 40 M**

9 (a) . State and Prove monotone convergence theorem.

(OR)

(b). Prove that the sequence $\{s_n\}$ defined by $s_n=1+\frac{1}{1!}+\frac{1}{2!}+\cdots+\frac{1}{n!}$ is convergent

10 (a) . State and prove Cauchy's n^{th} root test .

(OR)

(b) Examine the convergence of $\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$.

11 (a) If f is a continuous function on $[a,b]$, then prove that it is uniformly continuous on $[a,b]$

(OR)

(b) Test the continuity of of $f(x)=\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}\ if\ x\neq 0\ and\ f(0)=0$ at $x=0$.

12 (a) Show that $\frac{v-u}{1+v^2}<tan^{-1}v-\ tan^{-1}u<\frac{v-u}{1+u^2}\ for\ 0<u<v$. Hence deduce that $\frac{\pi}{4}+\frac{3}{25}<tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$.

(OR)

(b) State and Prove Cauchy's mean value theorem

13 (a) Show that $f(x)=3x+1$ is integrable on[1,2] and $\int_1^2(3x+1)dx=\frac{11}{2}$.

(OR)

(b) State and Prove first mean value theorem of integral calculus.